Current state of affairs – problem statement.

In the current MyPAM system the game (running at 30Hz) is responsible for generating the trajectory for each reaching movement. The game creates the final target position and generates equidistant intermediate positions as a linear path, as shown by figure x:

|  |
| --- |
|  |
| Figure x: An example of a linear trajectory generated by the game. |

These intermediate positions are passed 1 at a time to the controller. The controller (running at 500Hz) is only aware of the current position and the next intermediate position. Using position PID, the controller generates motor demands. A number of problems arise as a result of this current division of responsibilities and method:

1. A linear trajectory of this nature is not reflective of natural human motion.
2. The game does operate at 30 Hz reliably as a result of being dependant of the non-deterministic operating system on Windows. There may be instances where the game rate will drop, resulting in inaccurate intermediate position data being sent to the controller.
3. There are occasions where no intermediate points are generated and the final target position is sent to the controller, for example during some game types and during transitions between different games. This leads to a large difference between the current position and the target position, and large motor demands are generated. This leads to aggressive accelerations and potentially dangerous interaction forces between the patient and the robot.

Trajectory Generation

A smooth trajectory is desirable in order to mimic natural human motion when assisting the user to reach a target. Mathematically, a smooth trajectory translates to minimising the rate of change of an input, where the input corresponds to the order of the system. For example, a 1st order system corresponds to a kinematic model where velocities may be arbitrarily specified. This is summarised in the table x below:

|  |  |
| --- | --- |
| **Order of the system** | **Input to the system** |
| 1st | Velocity, |
| 2nd | Acceleration, |
| 3rd | Jerk, |
| 4th | Snap, |
| 5th | Crackle, |
| 6th | Pop, |

The function for the trajectory may be found using Calculus of Variations, using the general equation shown by Equation x.1:

|  |  |
| --- | --- |
|  | Eqn x.1 |

Where .

Alternatively, the trajectory may be found by satisfying the Euler-Lagrange equation shown by Equation x.2:

|  |  |
| --- | --- |
|  | Eqn x.2 |

For example, the shortest distance between 2 points may be found using Equation x.3 or x.4:

|  |  |
| --- | --- |
|  | Eqn x.3 |
|  | Eqn x.4 |

Calculating the Minimum Jerk Trajectory using the Euler-Lagrange formulation

For a Minimum Jerk Trajectory, , since a minimum jerk trajectory is based on minimising the sum of squared jerk across the trajectory.

Forming the Euler-Lagrange formulation as shown by Equation x.2:

|  |  |
| --- | --- |
|  | Eqn x.5 |
|  |  |
|  |  |
|  | Eqn x.6 |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  | Eqn x.7 |

Substituting boundary conditions as shown by table x:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Position | Velocity | Acceleration |
| t = 0 | a | 0 | 0 |
| T = tf | b | 0 | 0 |

As shown by Equation x.7:

|  |  |
| --- | --- |
|  | Eqn x.7 |

Thus:

|  |  |
| --- | --- |
|  | Eqn x.8 |
|  | Eqn x.9 |

Substituting boundary conditions:

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

Solving for the coefficients:

|  |  |
| --- | --- |
|  |  |
|  |  |

Giving:

|  |  |
| --- | --- |
|  | Eqn x.10 |
|  | Eqn x.11 |

Where:

Calculating the Minimum Jerk Trajectory using Calculus of Variations

For a Minimum Jerk Trajectory, . Using Equation x.1:

|  |  |
| --- | --- |
|  | Eqn x.1 |
|  |  |

Multiplying through by ½ tidies up the maths later on.

Introducing a small variation, , which has the following properties:

|  |  |
| --- | --- |
|  |  |

To minimise , add as a variation:

|  |  |
| --- | --- |
|  |  |
|  |  |

Differentiating with respect to :

|  |  |
| --- | --- |
|  |  |
|  |  |

Integrating by parts:

|  |  |
| --- | --- |
|  |  |

Where:

|  |  |
| --- | --- |
|  |  |

Thus:

|  |  |
| --- | --- |
|  |  |

Integrating by parts again:

|  |  |
| --- | --- |
|  |  |

Where:

|  |  |
| --- | --- |
|  |  |

Thus:

|  |  |
| --- | --- |
|  |  |

Integrating by parts a final time:

|  |  |
| --- | --- |
|  |  |

Where:

|  |  |
| --- | --- |
|  |  |

Thus:

|  |  |
| --- | --- |
|  |  |

Finally producing:

|  |  |
| --- | --- |
|  |  |

Since this must hold true for any function of which has the properties specified above, this means that Equation x.12 must be true:

|  |  |
| --- | --- |
|  | Eqn x.12 |

Which can then be solved for as shown in the previous section.

Justifying the use of a Minimum Jerk Trajectory

A Minimum Jerk Trajectory produces a smooth trajectory, but so too do other trajectories based on minimising the squared sum of derivatives of position. Figure number shows position and velocity graphs for the Minimum Acceleration, Minimum Jerk, Minimum Snap, Minimum Crackle and Minimum Pop trajectories to displace x from 0-100mm in 2 seconds. It may be observed that as the order of the system increases the position curve approaches a step function and the peak acceleration increases.

|  |
| --- |
|  |
| Figure Number: Position and Velocity Graphs showing a displacement of 100mm in 2 seconds using Minimum Acceleration, Minimum Jerk, Minimum Snap, Minimum Crackle and Minimum Pop trajectories. |

For assistive rehabilitation technology, it would be advantageous to generate a trajectory which mimics human movement. According to Richardson and Flash (2002), one measure of human reaching motion is the ratio of peak velocity to average velocity across the movement. Flash and Hogan (1985) found that the ratio of peak velocity to average velocity, R, across a reaching movement is around 1.8. The ratio for Minimum Acceleration, Minimum Jerk, Minimum Snap, Minimum Crackle and Minimum Pop trajectories is shown by table x:

|  |  |
| --- | --- |
| **Trajectory Type** | **R = Peak Velocity/Average Velocity** |
| Minimum Acceleration | 1.50 |
| Minimum Jerk | 1.88 |
| Minimum Snap | 2.19 |
| Minimum Crackle | 2.46 |
| Minimum Pop | 2.71 |

Thus, it is clear that by this measure the Minimum Jerk trajectory most closely resembles natural human motion and is therefore the most appropriate trajectory for the MyPAM.

VALIDATION FOR FUTURE WORK

Integrating a controller generated trajectory into the architecture

PIGGYBACK ONTO PPI THING WITH QUESTIONAIIRS

References

Richardson MJ, Flash T (2002) Comparing smooth arm movements with the two-thirds power law and the related segmented-control hypothesis. J Neurosci 22: 8201-8211.

Flash T, and Hogan N (1985) The coordination of arm movements: an experimentally confirmed mathematical model. J Neurosci 5: 1688-1703